

# Three-layer Permutation of the Phononic Structure and the Influence of the Environment

MICHAL SZOTA\*

Institute of Materials Engineering, ul. Armii Krajowej 19, 42-200 Czestochowa, Technical University of Czestochowa, Poland

The study investigated the influence of permutation of a three-layer structure on the phononic system transmission. Phononic structures are used as mechanical wave filters. These composites are designed and manufactured to have specific properties. The most important property is the presence of phononic bandgaps (PhBG) in these structures. They are designed so that the PhBG occurs in a given frequency range. Knowledge of the impact of deployment of the layers in the structure allows better design of these filters. The analysis was carried out using the transmission matrix method (TMM) algorithm. The transmission structures was examined for all permutations of a system of three layers made of different materials. The structure consisted of mercury, epoxy, rubber and PNM-0.38PT. The materials are chosen so that their characteristics largely differ. The structure was surrounded by water. The tests were carried out for the frequency range up to 1 MHz. Cases with different thickness of layers were analyzed. The tests have shown that regardless of the layer thickness, only three types of transmission structures exist in the six permutations of the system. Systems in which the middle layer remained unchanged, while the outermost layers were changed, were characterized by the same transmission structure. Increasing the thickness of the layers increased the number of transmission bands. Transmission strongly depends on the environment. The absorption of the materials used was not taken into account in the work. Interesting results can be obtained by analyzing the permutation of more complex structures. Changing the order of layers in the filter without changing its characteristics may affect the reduction of production costs and easier design of structures with given properties. The article shows repeating phononic transmission structure for different types of layers alignment.

**Keywords:** transmission, multilayers, superlattices, phononic, transfer matrix

The method of laying the layers in multilayer structures affects the formation of phononic bandgaps, which means that waves with certain frequencies do not propagate in the structure [1-2]. The structures in which this phenomenon occurs are used as acoustic devices which can be used as elastic/acoustic filters, waveguides or noise control [3-7]. Properly designed structures can be used as selective filters of acoustic wave or for demultiplexing [8, 9].

## Experimental part

For the analysis of phononic and photonic phenomena, among others, the Finite Difference Time Domain (FDTD) and Transfer Matrix Method (TMM) algorithms are used [10-29].

In the Transfer Matrix Method algorithm the acoustic wave propagation in multilayer structure can be described as:

$$\frac{1}{v_i^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = 0 \quad (1)$$

where:  $p$  is the pressure,  $v_i$  is  $i$ -layer phase velocity and  $t$  is a time.

In the quasi one-dimensional space, the solution of the equation takes the form

$$p_i = \left( A_i e^{i \frac{2\pi f}{v_i} x} + B_i e^{-i \frac{2\pi f}{v_i} x} \right) e^{-2\pi f t} \quad (2)$$

where:  $A_i$  is the amplitude of the transmitted wave and  $B_i$  is for a reflected one. As  $f$ , the frequency of the wave incident on the structure was determined. The main

equation of Transfer Matrix Method, where  $\Psi$  is a characteristic matrix, can be described as:

$$\begin{bmatrix} p_{i^+} & p_{i^-} \end{bmatrix}^T = \Psi \begin{bmatrix} p_{i+1}^+ & p_{i+1}^- \end{bmatrix}^T \quad (3)$$

where:

$$\Psi = TM_{n,n-1} \left( \prod_{i=1}^{n-1} PM_i TM_{i,i+1} \right) PM_n TM_{n,n+1} \quad (4)$$

The  $TM$  is a transfer matrix from layer  $i$  to  $j$  described as:

$$TM_{i,j} = \frac{1}{2} \begin{bmatrix} \frac{\rho_i v_i + \rho_j v_j}{\rho_i v_i} & \frac{\rho_i v_i - \rho_j v_j}{\rho_i v_i} \\ \frac{\rho_i v_i + \rho_j v_j}{\rho_i v_i} & \frac{\rho_i v_i - \rho_j v_j}{\rho_i v_i} \end{bmatrix} \quad (5)$$

where:  $\rho_i$  is density of mass of  $i$ -layer material and  $v_i$  is a phase velocity.

The propagation matrix  $PM$  in layer  $i$  is described as:

$$PM_i = \begin{bmatrix} e^{i \frac{2\pi f d_i}{v_i}} & 0 \\ 0 & e^{-i \frac{2\pi f d_i}{v_i}} \end{bmatrix} \quad (6)$$

for  $d_i$  layer thickness. Transmission is defined as:

$$T = Abs(\Psi_{11}^{-1})^2 \quad (7)$$

## Results and discussions

Materials with properties collected in table 1 were used for the analysis.

\*email: szota.michal@wip.pcz.pl

**Table 1**  
THE TYPES OF MATERIALS USED IN THE SIMULATION,  $\rho$   
IS MASS DENSITY,  $v$  IS PHASE VELOCITY

Symbol	Material	$\rho \left[ \frac{kg}{m^3} \right]$	$v \left[ \frac{m}{s} \right]$
A	Water	1000	1500
B	PNM-0.38PT	8093	4410
F	Epoxy	1180	2535
G	Mercury	13500	1450
H	Rubber	920	1550

The materials have been selected so that they differ significantly in physical properties. The structure was surrounded by water (material A). The layers were made of PNM-0.38PT, as material B, Epoxy as material F, Mercury as material G and Rubber as material H. The tests were carried out using the Transfer Matrix Method (TMM) algorithm. Lossless materials were analyzed. The permutations of a three-layer system of materials B, F and G were tested for the thickness of each layer equal to 5 mm (fig. 1) and 1 cm (fig. 2). Simulations were carried out for the range up to 1 MHz. In figures 1 and 2 the environment material was water.

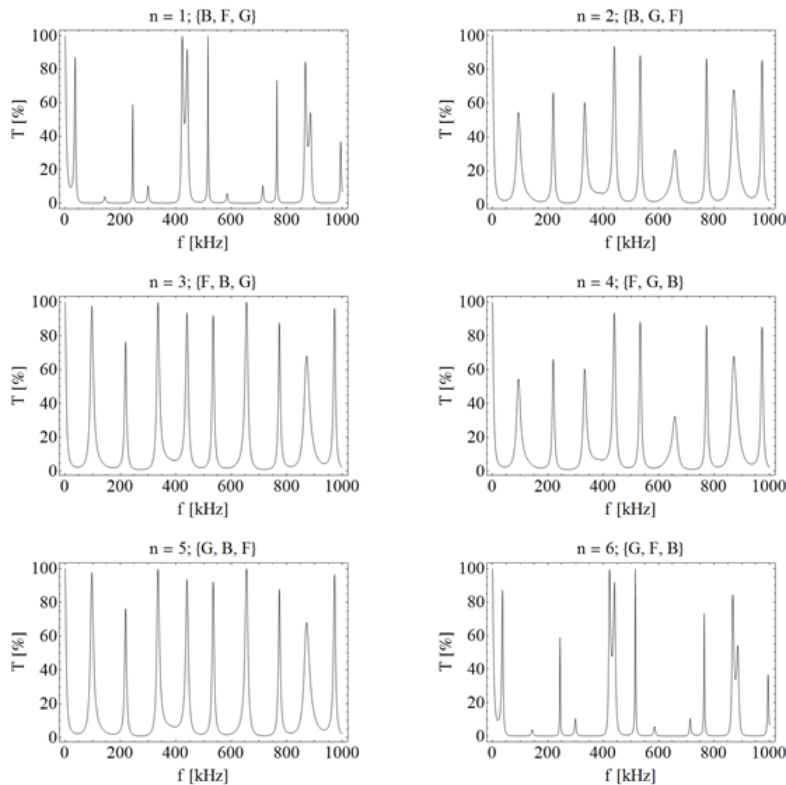


Fig. 1. Transmission maps for layer thickness equal to 5 mm for all permutations of three-layer structure made with G, F and B materials. The  $n$  is the permutation number with structure showed in curly brackets

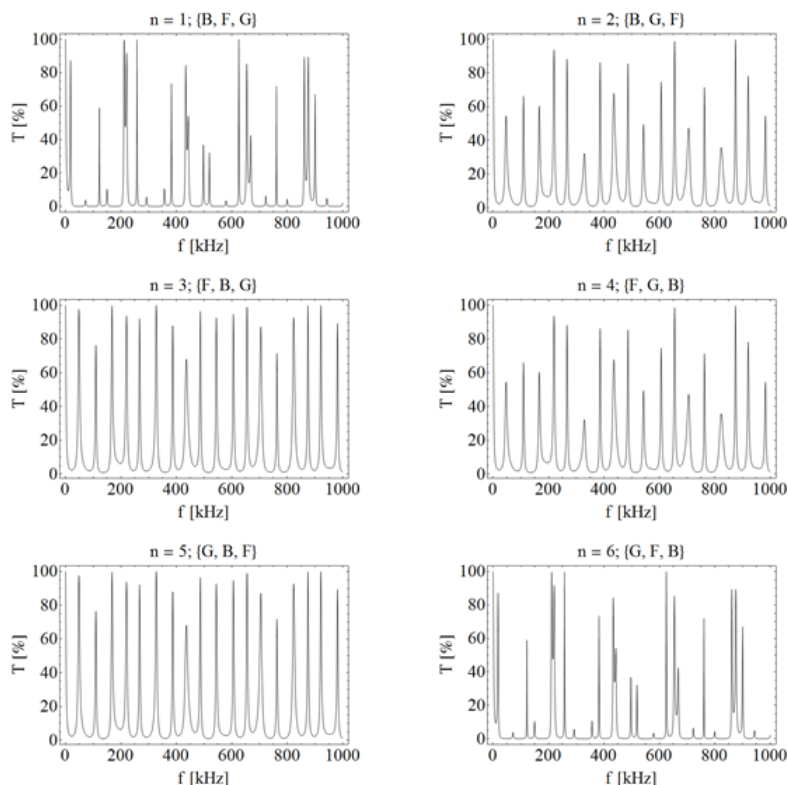


Fig. 2. Transmission maps for layer thickness equal to 1 cm for all permutations of three-layer structure made with G, F and B materials. The  $n$  is the permutation number with structure showed in curly brackets

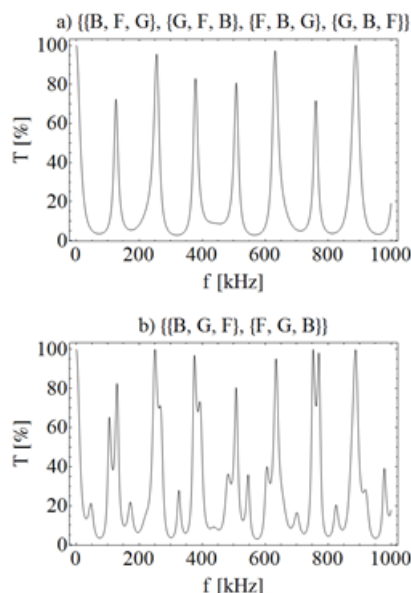


Fig. 3. The transmission for the permutations with the mercury environment. The thickness of the layers was 1 cm

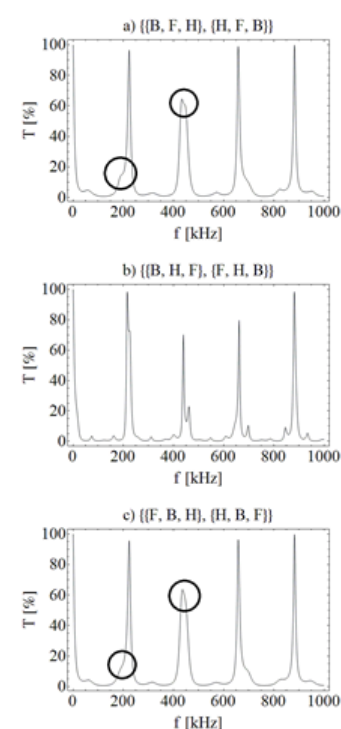


Fig. 4. The transmission for the permutations with the water environment. The thickness of the layers was 1 cm. Mercury has been replaced with rubber. Marked by circles on the graph the difference in transmission

From the analysis of the results, it can be noticed that in the transmission of the sound wave of the three-layer structure, there are alternate transmission peaks and phononic bandgaps (PnBG). Doubling of the thickness resulted in a significant increase the number of peaks. It should be noted that the structure with the numbers  $n$  equal to 1 and 6 had the same transmission structure despite different layering of the layers. Layer structure  $\{B, F, G\}$  and  $\{G, F, B\}$  respectively (figs. 1 and 2). The same transmission structure also had graphs for  $n$  equal to 3 and 5 ( $\{F, B, G\}$  and  $\{G, B, F\}$ ) and for  $n$  equal 2 and 4 ( $\{B, G, F\}$  and  $\{F, G, B\}$ ). It should be noted that identical transmission structures were for cases where the central layer remained unchanged, while the outer layers were changed in places.

The change of the material surrounding the structure from water to mercury resulted in only two transmission structures (fig. 3). With the change in the environment, the nature of the transmission has changed.

Figure 4 shows the transmission for a three-layer system where in comparison to figure 2 mercury has been replaced with rubber. The rubber properties similar to water caused a very similar transmission structure in diagrams

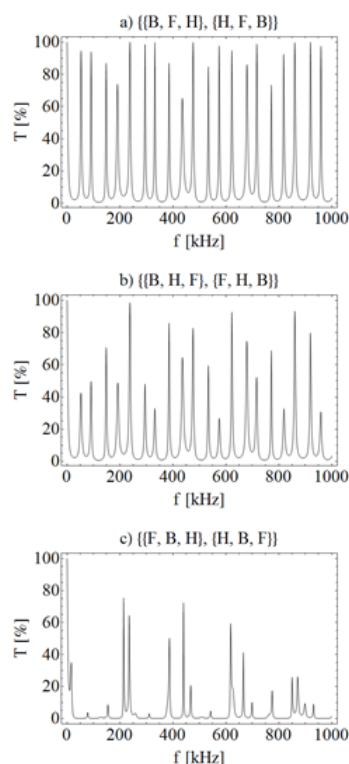


Fig. 5. The same structure as in figure 4 surrounded by mercury

4a and 4c, the differences were marked with circles. Converting mercury into rubber caused a significant reduction in the number of transmission peaks compared to figure 2.

Figure 5 shows the transmission of the same structure as in figure 4 only in the mercury environment instead of water. It should be noted that there is a clear difference between the three types of transmission for the tested structures and the re-occurrence of the transmission symmetry with respect to the middle layer.

## Conclusions

In the analyzed structures there were transmission peaks separated by bandgaps. The increase in layer thickness caused an increase in the number of transmission peaks. The structures in which the same middle layer occurred and the external layers were replaced were characterized by the same transmission distribution. Changing the type of one material in multilayer resulted in a significant change in the transmission structure. The environmental material had a very large impact on the filtering properties of the structure.

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